A Pandey, K K Singh and S, N. Dwivedi

On the Distribution of Interior Birth Intervals

1. Introduction

AN interior birth interval is a closed birth interval which begins and ends in any segment of age group or marriage duration. For instance, the intervals comprised by two successive births occurred in the segment of age group 25-35 or that occurred in between 10 and 20 years of marriage duration, are called interior birth intervals. It can also be described as a closed birth interval lying entirely between two survey dates as the start and end of the given time segment. Such type of closed birth intervals are usually obtained from the prospective enquiries started at time t_1 and continued till t_2, or from a retrospective enquiry conducted at time point t_2 and observe the birth intervals backward for last T years where $t_2 - t_1 = T$. And hence they have different distribution from usual closed birth intervals, indeed, the distribution of birth intervals often depends on the sampling frame under which they have been ascertained.

In recent past, for analysing interior birth intervals and studying their sensitivity in measuring fertility Poole (1971), Sheps and Menken (1972) and Menken and Sheps (1972) have tried to obtain some general distributions on the basis of the results of the simulated conception intervals where for application purpose the explicit form of the distribution was required. Bhattacharya and Singh (1982) have derived an explicit probability model for interior birth interval under the assumption that the birth densities do not change over time. However, if the birth densities do change over time, say after the start of the time segment (t_1 t_2) of length T in which the interior birth intervals are to be studied, the distribution of birth intervals will have some further form.

The objective of the present paper, hence has been to derive a probability distribution for interior birth intervals with a provision that the renewal (birth) densities change over time due to the use of some contraceptives, by a proportion of women in a community, of which the effectiveness has to be measured. The distribution is derived in Section 2 and its application is discussed in Section 3.

2. The Distribution

Let $X_i(T)$ denote ihe length of the interval between the ith and (i + 1)th births $(i \geq 1)$ occurring during the period (t_1, t_2) of length T in the reproductive life of a couple, where t_1 is a distant point since marriage, then treating time to be continuous the probability density function (p.d.f) of $X_i(T)$ has been obtained under the following assumptions:

(I) The woman under study has led a married life from the date of consummation of marriage to the end,of the observational period i e., t_2.

(II) The waiting time for first conception since marriage follows an exponential distribution with p.d.f.

$$f_1(t) = m\, e^{-mt}, \qquad t > 0,\ m > 0, \tag{2.1}$$

and the time between ith and $(i+1)$th conception follows a displaced exponential distribution with p.d.f.

$$f_2(t) = m\, e^{-m(t-h)}, \qquad t > h,\ m > 0, \tag{2.2}$$

where 'm' is called the conception rate and 'h' is the period of non-susceptibility comprised by the gestation period (g) and the period of post-partum amcnorrhea (PPA).

(III) There is one to one correspondence between a conception and a live birth.

(IV) A woman with probability *'a'* uses some contraceptives so that the risk parameter is suppressed and becomes $m^* = (1 - e)\, m$, where 'e' is the effectiveness of the contraceptiye.

If we denote by $g_i(x/T)$ the p.d.f. of $X_i(T)$, then under the above assumptions, we have

$$\begin{aligned} g_i(x/T) &= \frac{a\, g^*_{i1}(x/T)}{F^*_{i+1}(T)} + \frac{(1-a)\, g_{i1}(x/T)}{F_{i+1}(T)}, \quad h < x < T - i\,h \\ &= \frac{a\, g^*_{i2}(x/T)}{F^*_{i+1}(T)} + \frac{(1-a)\, g_{i2}(x/T)}{F_{i+1}(T)}, \\ &\qquad\qquad T - ih < x < T - g - \overline{i-1}\,h \\ &= \frac{a\, g^*_{i3}(x/T)}{F^*_{i+1}(T)} + \frac{(1-a)\, g_{i3}(x/T)}{F_{i+1}(T)}, \\ &\qquad\qquad T - g - \overline{i-1}\,h < x < T - \overline{i-1}\,h, \end{aligned} \tag{2.3}$$

where

$$g_{i1}(x/T) = m\, e^{-m(x-h)} + \frac{m}{1+mh} e^{-m(T-ih)} \sum_{s=0}^{i-2} \sum_{k=0}^{s} \frac{\{m(T-\overline{i-1}\,h-x)\}^k}{k\,!}$$

$$- \frac{m}{1+mh} e^{-m(T-\overline{i+1}\,h)} \sum_{s=0}^{i-1} \sum_{k=0}^{s} \frac{\{m(T-ih-x)\}^k}{k\,!} \tag{2.4}$$

$$g^*_{i1}(x/T) = m\left(\frac{1+m^*h}{1+mh}\right) e^{-m^*(x-h)} + \frac{m}{1+mh} e^{-m^*(T-ih)}$$

$$\sum_{s=0}^{i-2} \sum_{k=0}^{s} \frac{\{m^*(T-\overline{i-1}\,h-x)\}^k}{k\,!}$$

$$- \frac{m}{1+mh} e^{-m^*(T-\overline{i+1}\,h)} \sum_{s=0}^{i-1} \sum_{k=0}^{s} \frac{\{m^*(T-ih-x)\}^k}{k\,!}$$

$$- \frac{m-m^*}{1+mh} e^{-m^*(x-h)} + \frac{m-m^*}{1+mh} e^{-m^*(T-g-ih)}$$

$$\sum_{s=0}^{i-1} \frac{\{m^*(T-g-\overline{i-1}\,h-x)\}^s}{s\,!}; \tag{2.5}$$

$$g_{i2}(x/T) = \left\{\frac{m(T-x)}{1+mh} - (i-1)\right\} me^{-m(x-h)}$$

$$+ \frac{m}{1+mh} e^{-m(T-ih)} \sum_{s=0}^{i-2} \sum_{k=0}^{s} \frac{\{m(T-\overline{i-1}\,h-x)\}^k}{k\,!}; \tag{2.6}$$

$$g^*_{i2}(x/T) = \left\{\frac{m(T-x)}{1+mh} - (i-1)\frac{m(1+m^*h)}{m^*(1+mh)}\right\} m^* e^{-m^*(x-h)}$$

$$+ \frac{m}{1+mh} e^{-m^*(T-ih)} \sum_{s=0}^{i-2} \sum_{k=0}^{s} \frac{\{m^*(T-\overline{i-1}\,h-x)\}^k}{k\,!}$$

$$- \frac{m-m^*}{1+mh} e^{-m^*(x-h)} + \frac{m-m^*}{1+mh} e^{-m^*(T-g-ih)}$$

$$\sum_{s=0}^{i-1} \frac{\{m^*(T-g-\overline{i-1}\,h-x)\}^s}{s\,!}; \tag{2.7}$$

$$g_{ix}(x/T) = \left\{\frac{m(T-x)}{1+mh} - (i-1)\right\} m\, e^{-m(x-h)}$$

$$+ \frac{m}{1+mh} e^{-m(T-ih)} \sum_{s=0}^{i-2} \sum_{k=0}^{s} \frac{\{m(T-\overline{i-1}\,h-x)\}^k}{k\,!}; \quad (2.8)$$

$$g^*_{ix}(x/T) = \left\{\frac{m(T-x)}{1+mh} - (i-1)\,\frac{m(1+m^*h)}{m^*(1+mh)}\right\} m^*\, e^{-m^*(x-h)}$$

$$+ \frac{m}{1+mh}\; e^{-m^*(T-ih)} \sum_{s=0}^{i-2} \sum_{k=0}^{s} \frac{\{m^*(T-\overline{i-1}\,h-x)\}^k}{k\,!};$$

(2.9)

$$F_{i+1}(T) = 1 + \frac{1}{1+mh}\, e^{-m(T-ih)} \sum_{s=0}^{i-1} \sum_{k=0}^{s} \frac{\{m(T-ih)\}^k}{k\,!}$$

$$- \frac{1}{1+mh}\, e^{-m(T-\overline{i+1}\,h)} \sum_{s=0}^{i} \sum_{k=0}^{s} \frac{\{m(T-\overline{i-1}\,h)\}^k}{k} \quad (2.10)$$

$$F^*_{i+1}(T) = \frac{m(1+mh)}{m^*(1+m^*h)} + \frac{m}{m^*(1+mh)}\, e^{-m^*(T-ih)}$$

$$\sum_{s=0}^{i-1} \sum_{k=0}^{s} \frac{\{m^*(T-i\,h)\}^k}{k\,!} - \frac{m}{m^*(1+mh)}\, e^{-m^*}(T-\overline{i+1}\,h)$$

$$\sum_{s=0}^{i} \sum_{k=0}^{s} \frac{\{m^*(T-\overline{i+1}\,h)\}^k}{k\,!}$$

$$- \frac{m-m^*}{m^*(1+mh)}\left[1 - e^{-m^*}(T-g-ih) \sum_{s=0}^{i} \frac{\{m^*(T-g-ih)\}^s}{s\,!}\right].$$

(2.11)

Here $F_{i+1}(T)$ and $F^*_{i+1}(T)$ are the distribution functions corresponding to a density function which is ith fold convolution of the density function $f_2(t)$ itself and re-convolution with $f_1(t)$. For $a = 0$, the model derived by Bhattacharya and Singh (1982) for the above interval can be seen.

3. Application

The proposed distribution can be applied to observations relating to interior birth interval complied either for a longitudinal prospective survey or from different age segments of women's reproductive life under experiment of a contraceptive use effectiveness. The distribution involves four parameters *m*, *m**, *a* and *h*. The estimate or distribution of *h* can be had directly from a survey. Estimates of *m*, *m** and *a* can be obtained by the method of moments. The moments of the distribution will not be explicit with respect to the parameters *m* and *m** but may be explicit with respect to the parameter V. Hence, if we derive the expressions for first three moments, we can obtain the value of *'a'* in terms of *m* and *m** from first moment and substituting in the expressions of second and third moments we get two equations with two unknowns *m* and *m**. However, at this stage, these later two equations can be solved for *m* and *m** following the method of iteration. As such the difference between natural and controlled risk of conception as well as the proportion of users of the method under experiment can easily be ascertained. This distribution can repeatedly be used for finding out the fertility performance of women when they pass through different age groups and some of the women use some contraceptives during the segment of age under study.

References

1. Bhattacharra, B. N- and Singh, K. K., 1982, A Probability Model for Interior Birth Interval, Centre of Population Studies, B. H. U-, Varanasi (mimeo.)-
2. Menken, J. A. and Sheps, M. C.. 1972, Sampling Frame as a Determinant of Observed Distributions of Duration Variables, in : T. N. E. Greville (ed.), Population Dynamics, Academic Press, New York.
3. Poole, K., 1971, Fertility Based on Birth Interval Data, RTI Project No. 50-587. Research Triangle Park, N.C.
4. Shops, M. C. and Menken, J. A., 1972, Distribution of birth intervals according to the sampling frame, Theoretical Population Biology. 3(i): 1.